HAWKING RADIATION OF BLACK p-BRANES FROM GRAVITATIONAL ANOMALY

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We investigate the Hawking radiation of black p-branes of superstring theories using the method of anomaly cancelation, specially, we use the method of [S. Iso, H. Umetsu and F. Wilczek, *Phys. Rev. Lett.* **96**, 151302 (2006); *Phys. Rev. D* **74**, 044017 (2006)]. The metrics of black p-branes are spherically symmetric, but not the Schwarzschild type. In order to simplify the calculation, we first make a coordinate transformation to transform the metric to the Schwarzschild type. Then we calculate its energy-momentum flux from the method of anomaly cancelation of the above mentioned references. The obtained energy-momentum flux is equal to a black body radiation, the thermodynamic temperature of the radiation is equal to its Hawking temperature. And we find that the results are not changed for the original non-Schwarzschild type spherically symmetric metric.

Keywords: Hawking radiation; anomaly cancelation; black brane

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1 Introduction

Hawking radiation is an important physical property of black holes. Since its original discovery by Hawking,[1] many people have tried different methods for the derivation of this phenomenon.[2–6] The same result is obtained. Different methods for the derivation of Hawking radiation show that Hawking radiation is related with the quantum effect of the gravitational field of a black hole.

Recently, a new method for the derivation of Hawking radiation has been proposed by Robinson and Wilczek *et al.* [7,8] which is named anomaly cancelation. Robinson and Wilczek *et al.* [7,8] find that the effective field theory of quantum fields near a black hole's horizon is a two-dimensional chiral field theory due to the fact that a black hole's horizon is a one-way membrane. Thus there exist gauge and gravitational anomalies for the currents near a black hole's horizon. However the effective action of quantum fields near a black hole's horizon is still gauge invariant and general covariant. Then to combine the regular conditions for the covariantly anomalous currents on the horizon, gauge and energy-momentum fluxes with the Hawking temperature T_H are derived outside the horizon. In fact, the germination of this idea has appeared in a paper of Christensen and Fulling many years ago,[5] where the Hawking radiation of a $(1 + 1)$-dimensional Schwarzschild black hole was derived from the conformal anomaly method. The difference lies in that the primal method of Christensen and Fulling [5] is only applicable to $(1+1)$-dimensional spacetime manifold, while the method of Robinson and Wilczek *et al.* [7,8] can be applied to higher dimensional spacetime.

In Refs. 9–11, the method of anomaly cancelation for the derivation of Hawking radiation has been generalized to higher-dimensional rotating black holes. Then it has been used to the derivation of Hawking radiation of various types of black holes.[12–18] Some recent developments on this topic have been carried out in Refs. 19–21. In this paper, we study the Hawking radiation of general spherically symmetric black p-branes of superstring theories [22–26] using the method of anomaly cancelation. The reduced two-dimensional metrics of these black branes are still spherically symmetric, but not the Schwarzschild type generally. It is necessary to point out that in the original work of Robinson and Wilczek *et al*,[7,8] there are some differences between the method of Ref. 7 and Refs. 8. In Ref. 7, the Hawking flux of energy-momentum tensor is determined through canceling the gravitational anomaly in the consistent form at the horizon. In Refs. 8, the Hawking fluxes are determined by the conditions that the covariant current and energy-momentum tensor vanish at the horizon, instead of the consistent current.[11] For the Hawking radiation of non-Schwarzschild type spherically symmetric black holes from the method of anomaly cancelation, some results have been given by Ref. 17 using the method of Ref. 7. However, because the method of Refs. 8 is more widely used than the method of Ref. 7 for the calculation of Hawking fluxes of different types of black holes, as we can see from Refs. 10–16, etc., it is necessary for us to give a derivation of the Hawking radiation of general non-Schwarzschild type spherically

symmetric black holes to use the method of Refs. 8. Although some previous results on this problem have been given in Ref. 16, in this paper, we will give a different resolution for this problem.

In order to derive the Hawking radiation of general non-Schwarzschild type spherically symmetric black hole metrics from the method of anomaly cancelation of Refs. 8, we find that it is convenient to make a coordinate transformation to transform the metric to the form of the Schwarzschild type. This paper is organized as follows. In Sec. 2, we briefly review the metrics of black p-branes. In Sec. 3, we study the effective action of quantum fields near the horizon of a black brane. We obtain that it can be described by a two-dimensional field theory in a curved background. In Sec. 4, we make a coordinate transformation for the obtained non-Schwarzschild type two-dimensional spherically symmetric metric to the form of the Schwarzschild type. In Sec. 5, we calculate the energy-momentum flux related with the gravitational anomaly for the transformed two-dimensional metric. We obtain that the energy-momentum flux of a black p-brane obtained from the method of anomaly cancelation matches its Hawking radiation. Sec. 6 is the conclusion. In Appendix A, we derive the conditions for the Schwarzschild type coordinate transformation of Sec. 4.

2 The Metrics of Black p-Branes

In this section, we review the black p-brane solutions that appear in superstring theories.[22–26] We consider the following D-dimensional action

$$S = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(d+1)!} e^{-\alpha\phi} F_{d+1}^2 \right] , \tag{1}$$

where ϕ is a dilaton, F_{d+1} is the field strength for an antisymmetric tensor A_d of rank d, i.e.,

$$F_{d+1} = dA_d \ . \tag{2}$$

This action is a composition part of the low energy effective actions of type IIA and type IIB superstring theories. It is also a composition part of the action of eleven-dimensional supergravity, which is considered as the low energy effective action of M-theory.

The field equations of the action (1) have the following black $(d-1)$-brane solution:

$$\begin{aligned} ds^2 &= -\left[1-\left(\frac{r_+}{r}\right)^{\tilde{d}}\right]\left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{4\tilde{d}}{\delta(d+\tilde{d})}-1} dt^2 \\ &+ \left[1-\left(\frac{r_+}{r}\right)^{\tilde{d}}\right]^{-1}\left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{2\alpha^2}{\delta\tilde{d}}-1} dr^2 \\ &+ r^2\left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{2\alpha^2}{\delta\tilde{d}}} d\Omega_{\tilde{d}+1}^2 \end{aligned}$$

$$+ \quad \left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{4\tilde{d}}{\delta(d+\tilde{d})}} \delta_{ij}dx^i dx^j \ , \tag{3}$$

$$A_{01\ldots d-1} = \sqrt{\frac{4}{\delta}}\left(\frac{r_+ r_-}{r^2}\right)^{\frac{\tilde{d}}{2}} , \tag{4}$$

$$e^{-2\phi} = \left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{-\frac{4\alpha}{\delta}} , \tag{5}$$

where $i, j = 1, 2, \ldots, d-1$, $d\Omega_n^2$ is the metric of a unit n-sphere. In Eqs. (3)–(5), we have defined

$$\tilde{d} = D - d - 2 \ , \tag{6}$$

$$\alpha^2 = \delta - \frac{2d(D-d-2)}{D-2} \ . \tag{7}$$

The metric (3) has an event horizon at $r = r_+$, and an inner horizon at $r = r_-$, in condition that $r_+ > r_-$. The case of $r_+ = r_-$ corresponds to the extremal BPS state. In the following of this paper, we study the non-extremal case of this metric. We also use r_H to represent the radius of the event horizon in the following.

The Ramond-Ramond field strength for this solution is given by

$$e^{-\alpha\phi} * F_{d+1} = \sqrt{\frac{4}{\delta}}\tilde{d}(r_+ r_-)^{\frac{\tilde{d}}{2}}\epsilon_{\tilde{d}+1} \ , \tag{8}$$

where $\epsilon_{\tilde{d}+1}$ is the volume form on a unit $(\tilde{d}+1)$-sphere. The Ramond-Ramond charge with respect to the rank $(d+1)$ field strength that the black $(d-1)$-brane carries is

$$\begin{aligned} Q &= \frac{1}{16\pi}\int_{S^{\tilde{d}+1}} e^{-\alpha\phi} * F_{d+1} \\ &= \frac{1}{16\pi}\Omega_{\tilde{d}+1}\sqrt{\frac{4}{\delta}}\tilde{d}(r_+ r_-)^{\frac{\tilde{d}}{2}} \ , \end{aligned} \tag{9}$$

where Ω_n is the volume of a unit n-sphere.

The metrics of general spherically symmetric black branes including the metric (3) can be written in the form

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 C(r) d\Omega_{\tilde{d}+1}^2 + D(r)\delta_{ij}dx^i dx^j \ . \tag{10}$$

Their horizons are determined by $1/B(r)|_{r=r_H} = 0$. The Hawking temperature for the metric (10) can be obtained from the Euclidean approach. It is given by

$$\begin{aligned} T_H &= \frac{1}{2\pi}\frac{\partial_r\sqrt{A(r)}}{\sqrt{B(r)}}\bigg|_{r=r_H} \\ &= \frac{1}{4\pi}\frac{A'(r)}{\sqrt{A(r)B(r)}}\bigg|_{r=r_H} \ . \end{aligned} \tag{11}$$

For a general spherically symmetric black hole or black brane including the metric (3), its horizon is coincident with the surface of infinite red-shift, thus, $A(r)$ and $B(r)$ can be decomposed into

$$A(r) = a(r)b(r)\ , \qquad B(r) = \frac{c(r)}{a(r)}\ , \tag{12}$$

where

$$a(r_H) = 0 \tag{13}$$

determines the radius of the horizon, as well as the radius of the surface of infinite red-shift; but $b(r_H) \neq 0$, $c(r_H) \neq 0$. Here, we have $A(r)B(r) = b(r)c(r) \neq 1$ generally.

To substitute Eqs. (12) in Eq. (11), and to consider Eq. (13), the term that comes from the derivative of $b(r)$ vanishes. This yields

$$T_H = \frac{1}{4\pi}\left(a'(r)\sqrt{\frac{b(r)}{c(r)}}\right)\Bigg|_{r=r_H}\ . \tag{14}$$

For the metric (3), we have

$$\begin{aligned} a(r) &= 1 - \left(\frac{r_+}{r}\right)^{\tilde{d}}\ , \\ b(r) &= \left[1 - \left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{4\tilde{d}}{\delta(d+\tilde{d})}-1}\ , \\ c(r) &= \left[1 - \left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{2\alpha^2}{\delta\tilde{d}}-1}\ . \end{aligned} \tag{15}$$

To substitute Eqs. (15) in Eq. (14), or from Eq. (11), we obtain, for the metric (3),

$$T_H = \frac{\tilde{d}}{4\pi r_+}\left[1 - \left(\frac{r_-}{r_+}\right)^{\tilde{d}}\right]^{\frac{2\tilde{d}}{\delta(d+\tilde{d})}-\frac{\alpha^2}{\delta\tilde{d}}}\ . \tag{16}$$

To make use of Eqs. (6) and (7), we obtain, at last,

$$T_H = \frac{\tilde{d}}{4\pi r_+}\left[1 - \left(\frac{r_-}{r_+}\right)^{\tilde{d}}\right]^{\frac{2}{\delta}-\frac{1}{\tilde{d}}}\ . \tag{17}$$

The chemical potential with respect to the Ramond-Ramond charge is the value of the gauge potential at the horizon, i.e.,

$$\mu = A_{01\ldots d-1}|_{r=r_H} = \sqrt{\frac{4}{\delta}}\left(\frac{r_-}{r_+}\right)^{\frac{\tilde{d}}{2}}\ . \tag{18}$$

The first law of thermodynamics is satisfied by the black $(d-1)$-branes.

3 Quantum Fields Near The Horizon

Robinson and Wilczek *et al.* have shown that quantum fields near a black hole horizon can be described by a two-dimensional field theory in a curved background of the Schwarzschild type.[7,8] This is the primal recipe for the derivation of Hawking radiation from the method of anomaly cancelation. Thus, in order to study the Hawking radiation of black p-branes from the method of anomaly cancelation, we first need to study the effective field theory of quantum fields near a black brane horizon.

We consider a free scalar field φ in the background of the metric (10). To be simple, we suppose that the scalar field is zero-mass, and we suppose that it does not couple with the Ramond-Ramond gauge field. Then, the action of this system is given by

$$\begin{aligned} S_{\text{free}}(\varphi) &= \int d^D x\sqrt{-g}\; g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \\ &= \int d^D x\sqrt{-g}\,[\partial_\mu(\varphi\partial^\mu\varphi) - \varphi\partial_\mu\partial^\mu\varphi]\ , \end{aligned} \tag{19}$$

where g is the determinant of the metric (10), and thus

$$\sqrt{-g} = r^{\tilde{d}+1}\sqrt{A(r)B(r)C^{\tilde{d}+1}(r)D^{d-1}(r)}\ . \tag{20}$$

In considering that $\sqrt{-g}$ only depends on the coordinate r, while does not depend on the other coordinates, to omit a surface term in the action, we obtain

$$S_{\text{free}}(\varphi) = -\int d^D x\sqrt{-g}\varphi(\partial_\mu\partial^\mu + \frac{1}{\sqrt{-g}}\partial_r\sqrt{-g}\partial^r)\varphi\ . \tag{21}$$

To take the near horizon limit $r \to r_H$, the second term of Eq. (21) can be omitted compared with the first term. This fact can be made clear through transforming to the radial "tortoise" coordinate as that of Refs. 7 and 8. On the other hand, in the near horizon limit, $\sqrt{-g}$ tends to a constant, thus, it can be moved outside the integral. Therefore, near the black brane horizon, the action is dominated by

$$S_{\text{free}}(\varphi) = -\sqrt{-g}|_{r_H}\int dtdrd\Omega_{\tilde{d}+1}d^{d-1}x\ \varphi(\partial_t\partial^t + \partial_r\partial^r + \partial_{[D-2]}\partial^{[D-2]})\varphi\ , \tag{22}$$

where $d\Omega_{\tilde{d}+1}$ is the volume element of the $(\tilde{d}+1)$-dimensional unit sphere, $d^{d-1}x$ is the volume element of the $(d-1)$-dimensional transverse space. Here, we use $\partial_{[D-2]}\partial^{[D-2]}$ to represent the full Laplacian on the $(D-2)$-dimensional space whose explicit form is given by

$$\partial_{[D-2]}\partial^{[D-2]} = \frac{1}{r^2C(r)}\nabla^2_\Omega + \frac{1}{D(r)}\nabla^2_X\ , \tag{23}$$

where ∇^2_Ω is the Laplacian on the $(\tilde{d}+1)$-dimensional unit sphere, ∇^2_X is the Laplacian on the $(d-1)$-dimensional transverse space.

We can expand $\varphi(x)$ in terms of the normalized eigenfunctions of the operator $\partial_{[D-2]}\partial^{[D-2]}$. Therefore we have

$$\varphi(x) = \sum_{l_1,\dots,l_{\tilde{d}+1},k_1,\dots,k_{d-1}} \varphi_{l_1\dots l_{\tilde{d}+1}k_1\dots k_{d-1}}(r,t)Y_{l_1\dots l_{\tilde{d}+1}}(\theta_1,\dots,\theta_{\tilde{d}+1})X(k_1,\dots,k_{d-1}) \ . \tag{24}$$

In Eq. (24), $Y_{l_1\dots l_{\tilde{d}+1}}(\theta_1,\dots,\theta_{\tilde{d}+1})$ are the normalized spherical harmonics on a $(\tilde{d}+1)$-dimensional unit sphere with the azimuthal angles $(\theta_1,\dots,\theta_{\tilde{d}+1})$, and

$$X(k_1,\dots,k_{d-1}) = e^{i(k_1x^1+\dots+k_{d-1}x^{d-1})} \ , \tag{25}$$

they satisfy

$$\begin{aligned}\int d\Omega_{\tilde{d}+1}d^{d-1}xY_{l_1\dots l_{\tilde{d}+1}}(\theta_1,\dots,\theta_{\tilde{d}+1})X(k_1,\dots,k_{d-1}) \times \\ Y_{m_1\dots m_{\tilde{d}+1}}(\theta_1,\dots,\theta_{\tilde{d}+1})X^*(p_1,\dots,p_{d-1}) = \delta_{l_1m_1}\cdots\delta_{l_{\tilde{d}+1}m_{\tilde{d}+1}}\delta_{k_1p_1}\cdots\delta_{k_{d-1}p_{d-1}} \ ,\end{aligned} \tag{26}$$

where the integration of the coordinates x^i takes a unit volume on the transverse space of the brane.

In Eq. (24), $\varphi_{l_1\dots l_{\tilde{d}+1}k_1\dots k_{d-1}}(r,t)$ and $Y_{l_1\dots l_{\tilde{d}+1}}(\theta_1,\dots,\theta_{\tilde{d}+1})$ are real functions. However, because $X(k_1,\dots,k_{d-1})$ given by Eq. (25) are complex functions, $\varphi(x)$ given by Eq. (24) has been complexified. This means that in Eq. (22) inside the integral, the first φ should be replaced by φ^*. Thus, to substitute Eq. (24) in Eq. (22), the near horizon effective action of the free scalar field is obtained as

$$S_{\rm free}(\varphi) = -\sqrt{-g}|_{r_H} \sum_{l_1,\dots,l_{\tilde{d}+1},k_1,\dots,k_{d-1}} \int dtdr\varphi_{l_1\dots l_{\tilde{d}+1}k_1\dots k_{d-1}}(r,t)(\partial_t\partial^t+\partial_r\partial^r)\varphi_{l_1\dots l_{\tilde{d}+1}k_1\dots k_{d-1}}(r,t) \ . \tag{27}$$

From Eq. (27), we can see that, near the black brane horizon, the effective field theory of the free scalar field is a two-dimensional field theory in a curved background with the metric

$$ds^2 = -A(r)dt^2 + B(r)dr^2 \ . \tag{28}$$

The metric (28) is spherically symmetric, but not the Schwarzschild type generally, because $A(r)B(r) \neq 1$ generally. We can expect that for the other fields near the black brane horizon, their effective field theories are also two-dimensional field theories in a curved background of the metric (28). The explicit form of the reduced two-dimensional metric of the metric (3) is given by

$$\begin{aligned} ds^2 = & - \left[1-\left(\frac{r_+}{r}\right)^{\tilde{d}}\right]\left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{4\tilde{d}}{\delta(d+\tilde{d})}-1} dt^2 \\ & + \left[1-\left(\frac{r_+}{r}\right)^{\tilde{d}}\right]^{-1}\left[1-\left(\frac{r_-}{r}\right)^{\tilde{d}}\right]^{\frac{2\alpha^2}{\delta\tilde{d}}-1} dr^2 \ . \end{aligned} \tag{29}$$

If the scalar field carries the Ramond-Ramond charge, then it will couple with the Ramond-Ramond gauge field of the black brane. However, because the explicit forms of the covariant couplings of quantum fields with the Ramond-Ramond gauge fields are not clear, we do not consider such a coupling here in this paper; therefore, we will not study the Ramond-Ramond charge flux related with the gauge anomaly in the following.

4 Schwarzschild Type Coordinate Transformation

To use the metric (28) directly to calculate the Hawking flux using the method of Refs. 8 is not convenient for the case $A(r)B(r) \neq 1$. In order to simplify the calculation, we can make a coordinate transformation to transform the metric (28) to the Schwarzschild type first. They relate with the coordinates (t, r) through the transformation

$$t = t(\tau, r^*) \ , \qquad r = r(\tau, r^*) \ . \tag{30}$$

We demand that under this coordinate transformation, the metric (28) transforms to the form

$$ds^2 = -\sqrt{\frac{A(r^*)}{B(r^*)}} d\tau^2 + \sqrt{\frac{B(r^*)}{A(r^*)}} dr^{*2} \ , \tag{31}$$

which is now the Schwarzschild type. Here, we mean that the expression of the function $A(r^*)$ is the same as the expression of the function $A(r)$, the expression of the function $B(r^*)$ is the same as the expression of the function $B(r)$. The partial differential equations that the transformation (30) should satisfy are derived in Appendix A. Here, we only point out that such a kind of coordinate transformation is existing, it is also not unique; however, we need not to obtain its explicit form.

We can write Eq. (31) in the following form:

$$ds^2 = -F(r^*)d\tau^2 + \frac{1}{F(r^*)} dr^{*2} \ , \tag{32}$$

where

$$F(r^*) = \sqrt{\frac{A(r^*)}{B(r^*)}} \ . \tag{33}$$

As postulated above, the expression of the function $A(r^*)$ is the same as the expression of the function $A(r)$, the expression of the function $B(r^*)$ is the same as the expression of the function $B(r)$. Thus, according to Eqs. (12), the functions $A(r^*)$ and $B(r^*)$ can be decomposed into the form

$$A(r^*) = a(r^*)b(r^*) \ , \qquad B(r^*) = \frac{c(r^*)}{a(r^*)} \ , \tag{34}$$

where the expressions of the functions $a(r^*)$, $b(r^*)$, and $c(r^*)$ are the same as the expressions of the functions $a(r)$, $b(r)$, and $c(r)$ respectively. To substitute Eqs. (34) in Eq. (33), we can write

$$F(r^*) = a(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}} \ . \tag{35}$$

Then the condition

$$a(r_H^*) = 0 \tag{36}$$

determines the radius of the horizon of the metric (31) or (32), but $b(r_H^*) \neq 0$, $c(r_H^*) \neq 0$. However, because the expression of the function $a(r^*)$ is the same as the expression of the function $a(r)$, from Eqs. (36) and (13), we have

$$r_H^* = r_H \ , \tag{37}$$

which means that the location of the horizon of the metric (31) or (32) is the same as the location of the horizon of the metric (28), i.e., the location of the horizon is not changed after the Schwarzschild type coordinate transformation (30).

For the Schwarzschild type metric (32), its Hawking temperature is given by

$$T_H^* = \frac{1}{4\pi} F'(r^*)|_{r^*=r_H^*} \ . \tag{38}$$

To substitute Eq. (35) in Eq. (38), and to consider that on the horizon $a(r_H^*) = 0$, we obtain

$$T_H^* = \frac{1}{4\pi}\left(a'(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}} \right)\Bigg|_{r^*=r_H^*} \ . \tag{39}$$

As mentioned above, the expressions of the functions $a(r^*)$, $b(r^*)$, and $c(r^*)$ are the same as the expressions of the functions $a(r)$, $b(r)$, and $c(r)$ respectively, and because of Eq. (37), to compare Eq. (39) with Eq. (14), we have

$$T_H = T_H^* \ , \tag{40}$$

which means that the Hawking temperature of the metric (32) is the same as the Hawking temperature of the metric (28). The Hawking temperature of the metric (28) is not changed after the coordinate transformation (30). For the two-dimensional metric of Eq. (29), when transformed to the form of Eq. (32), its explicit form is given by

$$\begin{aligned} ds^2 &= -\left[1 - \left(\frac{r_+}{r^*}\right)^{\tilde{d}}\right]\left[1 - \left(\frac{r_-}{r^*}\right)^{\tilde{d}}\right]^{\frac{2\tilde{d}}{\delta(d+\tilde{d})} - \frac{\alpha^2}{\delta\tilde{d}}} d\tau^2 \\ &+ \left[1 - \left(\frac{r_+}{r^*}\right)^{\tilde{d}}\right]^{-1}\left[1 - \left(\frac{r_-}{r^*}\right)^{\tilde{d}}\right]^{\frac{\alpha^2}{\delta\tilde{d}} - \frac{2\tilde{d}}{\delta(d+\tilde{d})}} dr^{*2} \ . \end{aligned} \tag{41}$$

Here, the radius of the event horizon is $r_H^* = r_+$. It is not changed under the coordinate transformation (30). The radius of the inner horizon is r_-. It is not changed under the coordinate transformation (30) either. These facts can be seen from the metric (41). For the metric (41), its Hawking temperature can be obtained from Eq. (38) or (39) which is

$$T_H^* = \frac{\tilde{d}}{4\pi r_+}\left[1-\left(\frac{r_-}{r_+}\right)^{\tilde{d}}\right]^{\frac{2}{\delta}-\frac{1}{\tilde{d}}}. \tag{42}$$

It is just equal to the Hawking temperature of the metric (29) given by Eq. (17).

From the above analysis, we can see that, to perform a coordinate transformation (30), the metric (28) can be transformed to the form of Eq. (31) or (32), which is the Schwarzschild type. For a black hole or a black brane, because it is a thermal equilibrium system, its Hawking radiation only relies on its Hawking temperature and horizon's location in fact. However, for the metric (31) or (32), we have seen that its Hawking temperature and horizon's location are just the same as the Hawking temperature and horizon's location of the metric (28). This means that we can use the metric (31) or (32) equivalently to calculate the Hawking flux for the metric (28) using the method of anomaly cancelation.

5 Gravitational Anomaly And Energy-momentum Flux

In this section, we calculate the energy-momentum flux for the metric (32) using the method of anomaly cancelation. We will turn back to the metric (28) equivalently at last. Following Refs. 8, to consider the area outside the horizon, we divide it into two parts: $[r_H^*, r_H^*+\epsilon]$ and $[r_H^*+\epsilon, \infty]$. $[r_H^*, r_H^*+\epsilon]$ is the near horizon part, where the physics has certain exotic properties. $[r_H^*+\epsilon, \infty]$ is the part departed from the horizon, where the physics has the usual properties. The parameter ϵ can be taken arbitrarily small, thus, for the observably physical results, we can take them in the region $[r_H^*+\epsilon, \infty]$ always.

In the near horizon region $[r_H^*, r_H^*+\epsilon]$, to consider that a black hole's horizon is a one-way membrane, for the $(1+1)$-dimensional field theory, the ingoing (left moving) modes will tend to the center singularity, hence they will not affect the physics of the region $[r_H^*, r_H^*+\epsilon]$. That is to say in the region $[r_H^*, r_H^*+\epsilon]$, only the outgoing (right moving) modes are responsible for the observable physics. This makes the practical field theory be a two-dimensional chiral field theory in the near horizon region. Thus, in the region $[r_H^*, r_H^*+\epsilon]$, there exists the gravitational anomaly for the energy-momentum current. In the region $[r_H^*+\epsilon, \infty]$, the ingoing and outgoing modes are both existing, the field theory is a normal one while not chiral, there is no gravitational anomaly for the energy-momentum current.

For a black hole or a black brane, it is a thermodynamical equilibrium system, all currents in the spacetime outside the horizon are static. The energy-momentum tensor outside the

horizon can be decomposed into the form

$$T^{\mu}_{\nu}(r^*) = T^{\mu}_{\nu(H)}(r^*)H(r^*) + T^{\mu}_{\nu(o)}(r^*)\Theta_{+}(r^*) \ , \tag{43}$$

where $\Theta_+(r^*) = \Theta(r^* - r_+ - \epsilon)$ (here we use r_+ to represent the radius of the event horizon), and $H(r^*) = 1 - \Theta_+(r^*)$. Thus, $T^{\mu}_{\nu(H)}(r^*)$ is the energy-momentum tensor in the region $[r^*_H, r^*_H + \epsilon]$, $T^{\mu}_{\nu(o)}(r^*)$ is the energy-momentum tensor in the region $[r^*_H + \epsilon, \infty]$.

In a two-dimensional spacetime, $T^r_t(r^*)$ is just the energy-momentum flux in the radial direction. For $T^{\mu}_{\nu(o)}(r^*)$, as analyzed above, there is no gravitational anomaly, it satisfies the normal conservation equation, therefore we have

$$\partial_{r^*} T^r_{t(o)}(r^*) = 0 \ . \tag{44}$$

But for $T^{\mu}_{\nu(H)}(r^*)$ near the horizon, it has the gravitational anomaly. It satisfies the anomalous conservation equation.[7,8,27,28] This results

$$\partial_{r^*} T^r_{t(H)}(r^*) = \partial_{r^*} N^r_t(r^*) \ , \tag{45}$$

where the right hand side of Eq. (45) comes from the gravitational anomaly of the consistent energy-momentum tensor. For the two-dimensional metric (32), to use the result of Refs. 7 and 8, we have

$$N^r_t(r^*) = \frac{1}{192\pi}\Big[(F'(r^*))^2 + F''(r^*)F(r^*)\Big] . \tag{46}$$

The integration of Eqs. (44) and (45) yields

$$\begin{aligned} T^r_{t(o)}(r^*) &= a^*_o \ , \\ T^r_{t(H)}(r^*) &= a^*_H + \int_{r^*_H}^{r^*} dr^* \partial_{r^*} N^r_t(r^*) \ , \end{aligned} \tag{47}$$

where a^*_o, a^*_H are two integration constants. From Eq. (43), we can see that a^*_o is just the energy-momentum flux for an observer to measure outside the horizon.

On the other hand, the anomaly of the energy-momentum tensor is purely a quantum field effect, it does not affect the general covariance of the effective action. To perform an infinitesimal coordinate transformation for the two-dimensional field theory along the time direction with the parameter ξ^t, we obtain [8]

$$\begin{aligned} \delta W &= -\int d^2x\sqrt{-g}\ \xi^t \nabla_\mu T^\mu_t \\ &= -\int d^2x\ \xi^t \Big[\partial_r(N^r_t H) + \Big(T^r_{t(o)} - T^r_{t(H)} + N^r_t\Big)\delta(r^* - r_+ - \epsilon)\Big] \ . \end{aligned} \tag{48}$$

As pointed out in Refs. 8, the first term in the second line of Eq. (48) can be canceled by the quantum effect of the ingoing modes near the horizon. Thus, general covariance of the

effective action leads the vanishing of the coefficient of the δ-function. Then, to combine Eqs. (47), we obtain

$$a_o^* = a_H^* - N_t^r(r_H^*) \ . \tag{49}$$

Substituting Eq. (35) in Eq. (46), and considering that on the horizon $a(r_H^*) = 0$, we obtain

$$N_t^r(r_H^*) = \frac{1}{192\pi}\left(a'(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}}\right)^2\Bigg|_{r^*=r_H^*} . \tag{50}$$

In order to determine the constant a_H^* of Eq. (49), we need to introduce the covariantly anomalous energy-momentum tensor $\widetilde{T}_{\mu\nu}$, it satisfies the covariant anomaly equation[28]

$$\nabla^\mu \widetilde{T}_{\mu\nu} = \frac{1}{96\pi\sqrt{-g}}\epsilon_{\mu\nu}\partial^\mu R \ , \tag{51}$$

where R is the Ricci scalar. For the component $\widetilde{T}_t^r$ which is necessary for the following calculation, for the metric (32), to use the result of Refs. 8, we have

$$\widetilde{T}_t^r(r^*) = T_t^r(r^*) + \frac{1}{192\pi}\left[F(r^*)F''(r^*) - 2(F'(r^*))^2\right] . \tag{52}$$

As postulated in Refs. 8, $\widetilde{T}_t^r$ satisfies the regular boundary condition

$$\widetilde{T}_t^r(r_H^*) = 0 \ , \tag{53}$$

such a boundary condition makes physical quantities regular on the future horizon for a free falling observer.[8,9] The combination of Eqs. (52), (53), and (43) yields

$$T_{t(H)}^r(r_H^*) = \frac{1}{192\pi}\left[2(F'(r^*))^2 - F(r^*)F''(r^*)\right]\Big|_{r^*=r_H^*} . \tag{54}$$

Substituting for $F(r^*)$ from Eq. (35), and considering that on the horizon $a(r_H^*) = 0$, we obtain

$$T_{t(H)}^r(r_H^*) = \frac{1}{96\pi}\left(a'(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}}\right)^2\Bigg|_{r^*=r_H^*} . \tag{55}$$

Then, the combination of Eqs. (55) and (47) results

$$a_H^* = \frac{1}{96\pi}\left(a'(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}}\right)^2\Bigg|_{r^*=r_H^*} . \tag{56}$$

Substituting Eqs. (56) and (50) in Eq. (49), we obtain

$$a_o^* = \frac{1}{192\pi}\left(a'(r^*)\sqrt{\frac{b(r^*)}{c(r^*)}}\right)^2\Bigg|_{r^*=r_H^*} . \tag{57}$$

Finally, comparing Eq. (57) with Eq. (39), we can write

$$a_o^* = \frac{\pi}{12} T_H^{*2} \ , \tag{58}$$

where T_H^* is just the Hawking temperature of the metric (32). The explicit expression of T_H^* of the metric (41) is given by Eq. (42). As mentioned above, a_o^* is just the energy-momentum flux for an observer to measure outside the horizon of the metric (32). From the above derivation, we can see that the anomalous current a_H^* near the horizon, i.e., the outgoing chiral current, has contribution to the current a_o^* departed from the horizon which is a normal one, or, we can say that the existence of the outgoing chiral current makes the radiation current be a normal one and cancels its anomaly.

To consider a two-dimensional black body radiation with temperature T, the distribution of a zero-mass bose field is given by

$$N(\omega) = \frac{1}{e^{\omega/T} - 1} \ . \tag{59}$$

Here, we need not to consider the distribution of a fermion field in order to avoid the superradiance problem, because there is no superradiance for a non-rotating black hole or black brane. The energy-momentum flux of this two-dimensional black body radiation is obtained as

$$F_E = \frac{1}{2\pi} \int_0^\infty \omega N(\omega) d\omega = \frac{\pi}{12} T^2 \ . \tag{60}$$

To compare Eq. (58) with Eq. (60), we can see that Eq. (58) is just the energy-momentum flux of a two-dimensional black body radiation with the temperature T_H^*. Therefore, for the transformed two-dimensional Schwarzschild type metric (32), it has a black body radiation with the thermal temperature given by its Hawking temperature.

For a black hole or a black brane, to be a thermal equilibrium system, its Hawking radiation should only be determined by its Hawking temperature and horizon's location. For the metric (28), its Hawking temperature and horizon's location are just the same as the Hawking temperature and horizon's location of the metric (32), its Hawking temperature and horizon's location are not changed under the coordinate transformation (30), thus, we can deduce that, for the metric (28), it has a Hawking radiation as same as the Hawking radiation of the metric (32). This means that for the original non-Schwarzschild type spherically symmetric metric (28), we have, outside its horizon, a radial energy-momentum flux given by

$$a_o = \frac{\pi}{12} T_H^2 \ , \tag{61}$$

where T_H is just its Hawking temperature. Hence, we have derived that there is a Hawking radiation for the two-dimensional non-Schwarzschild type spherically symmetric metric (28) outside its horizon from the method of anomaly cancelation of Refs. 8, and the temperature

of the thermal radiation derived from the method of anomaly cancelation is in accordance with its Hawking temperature derived from the black brane thermodynamics.

There is a problem needing to be deliberated further. From Eqs. (47), $a_o^* = T^r_{t(o)}(r^*)$, at the same time, $T^r_{t(o)}(r^*)$ is a component of the second-order tensor $T^\mu_{\nu(o)}(r^*)$. For the energy-momentum flux $T^r_{t(o)}(r)$ of the original metric (28), which is a component of the second-order tensor $T^\mu_{\nu(o)}(r)$ of the original metric (28), we need to obtain it from the coordinate transformation (30) in principle, and it will not remain to be a constant of a_o^* generally because of the nonlinearity of the coordinate transformation. Thus it seems that we can not obtain the correct result for the Hawking flux of the original metric (28). However, such a problem is only superficial, it comes from the coordinate transformation in fact. This is because, first, for the metrics (28) and (32), or (29) and (41), they are all asymptotic flat. Therefore, at spacelike infinity, the coordinate transformation (30) and its inverse transformation will tend to an identical transformation necessarily. This means that for the original metric (28), at infinity, we have $T^r_{t(o)}(\infty) = a_o^*$. Next, for the tensor $T^\mu_{\nu(o)}(r)$ of the original metric (28), there is no gravitational anomaly, like that of Eq. (44), it satisfies the normal conservation equation. This results

$$\partial_r T^r_{t(o)}(r) = 0 \ , \tag{62}$$

whose solution is given by

$$T^r_{t(o)}(r) = a_o \ , \tag{63}$$

where a_o is a constant. But we have known from the above analysis that, at infinity, $T^r_{t(o)}(\infty) = a_o^*$. Then, from Eq. (63), we have

$$T^r_{t(o)}(r) = a_o^* \tag{64}$$

for an arbitrary r. Because a_o^* has been obtained in Eq. (58), we obtain, for the original metric (28), its energy-momentum flux is given by

$$a_o = \frac{\pi}{12} T_H^{*2} \ . \tag{65}$$

In considering Eq. (40), we can write

$$a_o = \frac{\pi}{12} T_H^2 \ , \tag{66}$$

where T_H is the Hawking temperature of the original metric (28). This means that the energy-momentum flux of the original two-dimensional metric (28) is a constant, and, it is equal to a two-dimensional black body radiation, the temperature of the radiation is just its Hawking temperature obtained from the black brane thermodynamics. For the original higher-dimensional metric (3) or (10), from the mode decomposition of the field in terms of Eq. (24), it is not difficult to see that the distribution of the spectrum will not change.

Therefore, for the black brane metric (3), we can conclude that it has a Hawking radiation with the temperature T_H from the method of anomaly cancelation.

However, if we do not perform the coordinate transformation (30) and use the original metric (28) directly to calculate the energy-momentum flux from the method of anomaly cancelation, then, for $\widetilde{T}^r_t(r)$, which is a component of the covariantly anomalous energy-momentum tensor $\widetilde{T}^\mu_\nu(r)$, it seems that there will be some additional factors of the form $\sqrt{-g_2} = \sqrt{A(r)B(r)}$, and they can not be canceled due to the metric is not the Schwarzschild type. Therefore, we constructed a coordinate transformation (30) to transform the metric to the Schwarzschild type first, and then, we use the method of anomaly cancelation of Refs. 8 to calculate its Hawking flux.

6 Conclusion

In this paper, we studied the Hawking radiation of black p-branes of superstring theories [22–26] from the method of anomaly cancelation of Refs. 8. The metrics of these black p-branes are spherically symmetric, but not the Schwarzschild type. The calculation of this paper are carried out with respect to the general non-Schwarzschild type spherically symmetric black hole metrics.

Some previous works have been taken for the Hawking fluxes of non-Schwarzschild type spherically symmetric black hole metrics using the method of anomaly cancelation.[16–18] In Ref. 18, Hawking radiation of the $D1$-$D5$ brane black holes of superstring theories have been calculated using the method of anomaly cancelation. The black hole metrics studied in Ref. 18 are non-Schwarzschild type; however, they belong to certain special kind of the metrics of general non-Schwarzschild type spherically symmetric black holes.

In Ref. 17, Hawking radiation are also calculated with respect to the general non-Schwarzschild type spherically symmetric black hole metrics. However, the method adopted in this paper is different from that of Ref. 17. In Ref. 17, Hawking radiation are calculated using the method of Ref. 7, i.e., the Hawking flux of energy-momentum tensor is determined through canceling the gravitational anomaly in the consistent form at the horizon. In this paper, we used the method of Refs. 8, i.e., the Hawking fluxes are determined by the conditions that the covariant current and energy-momentum tensor vanish at the horizon, instead of the consistent current. Because the method of Refs. 8 is more general than the method of Ref. 7 for the calculation of Hawking fluxes of different types of black holes, as we can see from Refs. 10–16, etc., it is necessary for us to study the Hawking radiation of general non-Schwarzschild type spherically symmetric black hole metrics to use the method of Refs. 8.

On the other hand, we can see that in Ref. 16, Hawking radiation of general non-Schwarzschild type spherically symmetric black hole metrics from the method of Refs. 8

has been analyzed to use the metric (28) directly, while not through transforming it to the Schwarzschild type. However, many formulas have become complicated because of this as we can see from Ref. 16. In order to give a clearer derivation for this problem, we find that it is convenient to make a coordinate transformation to transform the non-Schwarzschild type spherically symmetric black hole metric to the Schwarzschild type first, and then to calculate the energy-momentum flux for the transformed metric from the method of anomaly cancelation. The obtained energy-momentum flux is equal to a black body radiation, the thermodynamic temperature of the radiation is equal to the Hawking temperature. And we find that the results are not changed for the original non-Schwarzschild type spherically symmetric metric. Then, from the mode decomposition of the scalar field in terms of Eq. (24), we can deduce that the distribution of the spectrum is not changed for the whole higher-dimensional object. Thus, we have derived the Hawking radiation of black p-branes from the method of anomaly cancelation of Refs. 8. The result of this paper is held for a general higher-dimensional non-Schwarzschild type spherically symmetric black hole and black brane.

It is necessary to point out here that we have not studied the Ramond-Ramond charge fluxes of the black p-branes from the method of anomaly cancelation in this paper. If the scalar field carries the Ramond-Ramond charge, then it will couple with the Ramond-Ramond gauge field of the black brane. However, the explicit forms of the covariant couplings of quantum fields with the Ramond-Ramond gauge fields are different from that of the $U(1)$ gauge field. At the same time, the anomaly equations of the Ramond-Ramond charge currents are also different from that of the electric charge current. Therefore we cannot use the method of anomaly cancelation directly for the calculation of the Ramond-Ramond charge fluxes of the black p-branes. For such a problem, we hope to study it in the future.

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Appendix A. The Conditions for The Schwarzschild Type Coordinate Transformation

In this appendix, we derive the conditions for the Schwarzschild type coordinate transfor-

mation (30). The original two-dimensional metric is given by Eq. (28):

$$ds^2 = -A(r)dt^2 + B(r)dr^2 \ . \tag{A.1}$$

The unknown coordinate transformation is

$$t = t(\tau, r^*) \ , \qquad\qquad r = r(\tau, r^*) \ . \tag{A.2}$$

Therefore we have

$$dt = \frac{\partial t}{\partial \tau}d\tau + \frac{\partial t}{\partial r^*}dr^* \ , \tag{A.3}$$

$$dr = \frac{\partial r}{\partial \tau}d\tau + \frac{\partial r}{\partial r^*}dr^* \ . \tag{A.4}$$

We demand that under this coordinate transformation, the metric (A.1) transforms to the form of Eq. (31), i.e.

$$ds^2 = -\sqrt{\frac{A(r^*)}{B(r^*)}}d\tau^2 + \sqrt{\frac{B(r^*)}{A(r^*)}}dr^{*2} \ . \tag{A.5}$$

Here, we mean that the expression for the function $A(r^*)$ is the same as the expression for the function $A(r)$, the expression for the function $B(r^*)$ is the same as the expression for the function $B(r)$.

To substitute Eqs. (A.3) and (A.4) in Eq. (A.1), then, to compare with Eq. (A.5), we obtain the following equations:

$$-A(r)\left(\frac{\partial t}{\partial \tau}\right)^2 + B(r)\left(\frac{\partial r}{\partial \tau}\right)^2 = -\sqrt{\frac{A(r^*)}{B(r^*)}} \ , \tag{A.6}$$

$$-A(r)\left(\frac{\partial t}{\partial r^*}\right)^2 + B(r)\left(\frac{\partial r}{\partial r^*}\right)^2 = \sqrt{\frac{B(r^*)}{A(r^*)}} \ , \tag{A.7}$$

$$-2A(r)\frac{\partial t}{\partial \tau}\frac{\partial t}{\partial r^*} + 2B(r)\frac{\partial r}{\partial \tau}\frac{\partial r}{\partial r^*} = 0 \ . \tag{A.8}$$

From Eq. (A.4), we can consider that the function $r(\tau, r^*)$ is determined by $\frac{\partial r}{\partial \tau}$ and $\frac{\partial r}{\partial r^*}$. Thus, in equations (A.6)–(A.8), we can regard the function $r(\tau, r^*)$ not as an independent unknown function. In equations (A.6)–(A.8), the expressions for the functions $A(r)$, $B(r)$, $A(r^*)$, and $B(r^*)$ are known, the independent unknown functions are therefore $\frac{\partial r}{\partial \tau}$, $\frac{\partial r}{\partial r^*}$, $\frac{\partial t}{\partial \tau}$, and $\frac{\partial t}{\partial r^*}$. Hence there are four independent unknown functions and three independent equations in equations (A.6)–(A.8). We can expect that the solutions for $\frac{\partial r}{\partial \tau}$, $\frac{\partial r}{\partial r^*}$, $\frac{\partial t}{\partial \tau}$, and $\frac{\partial t}{\partial r^*}$ are existing, and they are not unique generally. Then, from the solutions of $\frac{\partial r}{\partial \tau}$, $\frac{\partial r}{\partial r^*}$, $\frac{\partial t}{\partial \tau}$, and $\frac{\partial t}{\partial r^*}$, we can obtain the solutions of $t(\tau, r^*)$ and $r(\tau, r^*)$. They are not unique generally. Therefore, such a kind of coordinate transformation is existing, and it is not unique generally. However, we need not to obtain its explicit form.

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